$R^2 \geq 0$

Pantelis Sopasakis

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Notes

The *total sum of squares*, the *explained sum of squares* and the *residual sum of squares* are defined respectively as follows:

$$SS_{tot} \triangleq \sum_{i \in \mathbb{I}} (y_i - \bar{y})^2 \tag{1}$$

$$SS_{reg} \triangleq \sum_{i \in \mathbb{I}} (f_i - \bar{y})^2 \tag{2}$$

$$SS_{err} \triangleq \sum_{i \in \mathbb{I}} (f_i - y_i)^2 \tag{3}$$

It follows from the triangle inequality that:

$$SS_{tot} \leq SS_{reg} + SS_{err} \tag{4}$$

Now R squared is defined as follows:

$$R^2 \triangleq 1 - \frac{SS_{err}}{SS_{tot}} = \frac{SS_{tot} - SS_{err}}{SS_{tot}} \tag{5}$$

So by means of (4), one has that:

$$R^2 \geq \frac{SS_{tot} - (SS_{tot} - SS_{reg})}{SS_{tot}} = \frac{SS_{reg}}{SS_{tot}} \geq 0 \tag{6}$$